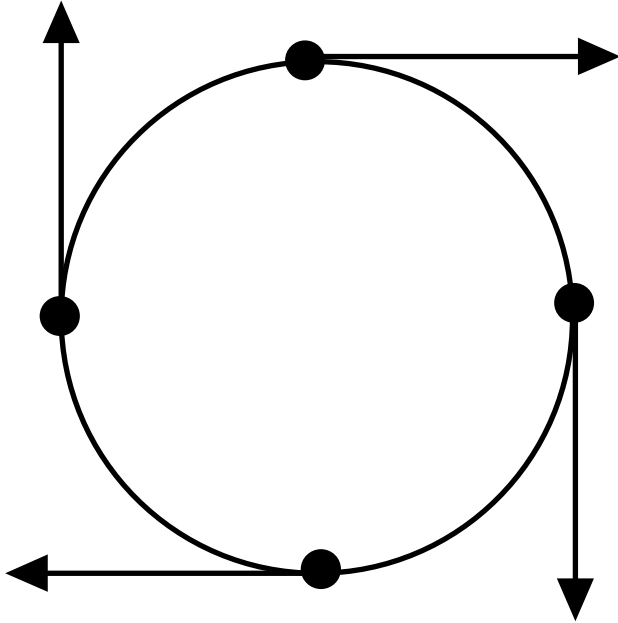
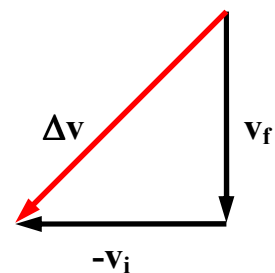
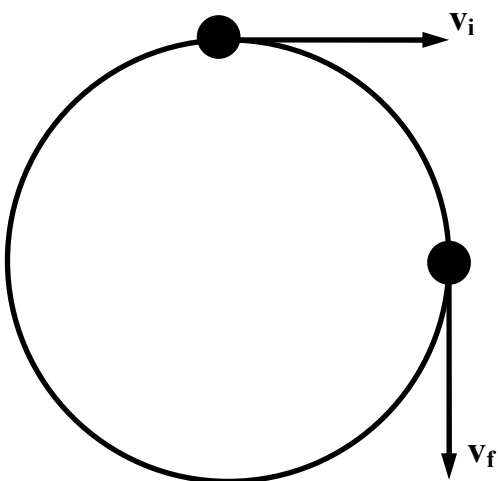


Centripetal Acceleration

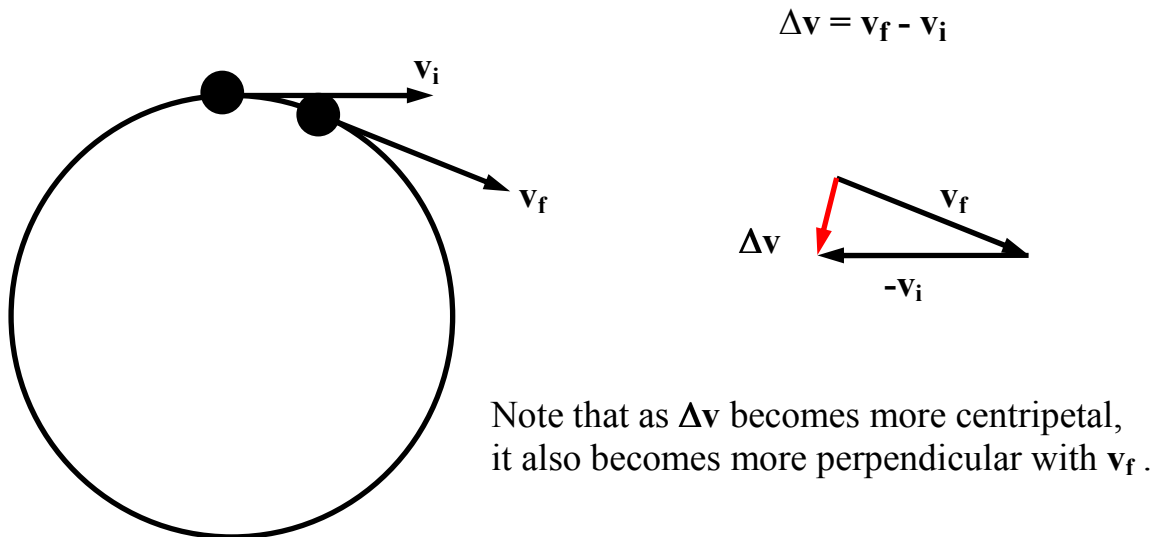
Imagine a marble sitting on a rotating turntable. The different vectors representing velocity for the travelling marble are shown below.



Notice that the size of the vector remains the same but the direction is constantly changing. Because the direction is changing, there is a $\Delta \mathbf{v}$ and $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$, and since velocity is changing, circular motion must also be *accelerated* motion.



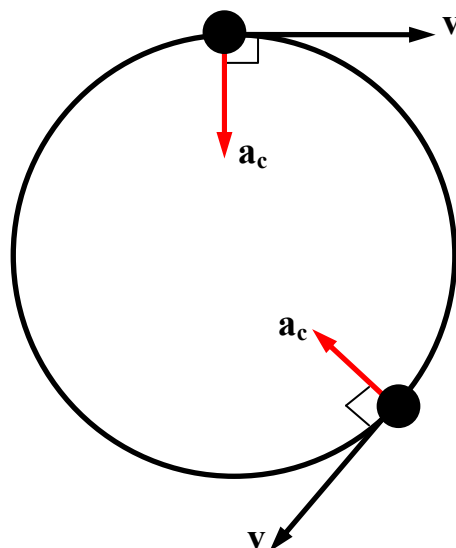
If the Δt in-between initial velocity and final velocity is small, the direction of $\Delta \mathbf{v}$ is nearly radial (i.e. directed along the radius). As Δt approaches 0, $\Delta \mathbf{v}$ becomes exactly radial, or *centripetal*.



Also note that the acceleration of an object depends on its change in velocity $\Delta \mathbf{v}$; i.e., if $\Delta \mathbf{v}$ is centripetal, so is ' \mathbf{a} '.

From this, we can conclude the following for any object travelling in a circle at constant speed:

- The velocity of the object is tangent to its circular path.
- The acceleration of the object is centripetal to its circular path. This type of acceleration is called *centripetal acceleration*, or \mathbf{a}_c .
- The centripetal acceleration of the object is always perpendicular to its velocity at any point along its circular path.



- To calculate the magnitude of the tangential velocity (i.e., the speed) of an object travelling in a circle:

- Start with $d = v_{av}t$ where ' v_{av} ' is a constant speed ' v '
- In a circle, distance = circumference, so $d = 2\pi r$
- The time ' t ' taken to travel once around the circular path is the object's period ' T '
- Therefore, the object's speed is

$$v = \frac{2\pi r}{T}$$

Note that frequency $f = \frac{1}{T} \rightarrow$ therefore $v = 2\pi r f$

- There are two formulas for calculating the object's centripetal acceleration:
- In terms of the object's speed and radius of circular path:

$$a_c = \frac{v^2}{r}$$

- In terms of the object's period and radius of circular path, take the above formula and substitute in $v = 2\pi r/T$ to produce the following:

$$a_c = \frac{4\pi^2 r}{T^2}$$

- Note that all values are measured in SI units; e.g., centripetal acceleration is measured in m/s^2 .

Example #1: A child stands on a merry-go-round that is spun around with a period of 2.3 s.

- If the child is 0.65 m from the center of the merry-go-round, what is her acceleration?**
- If she then moves to a distance of 1.4 m from the center, at what speed will she be travelling?**

(see Circular Motion Ex 1 for answer)

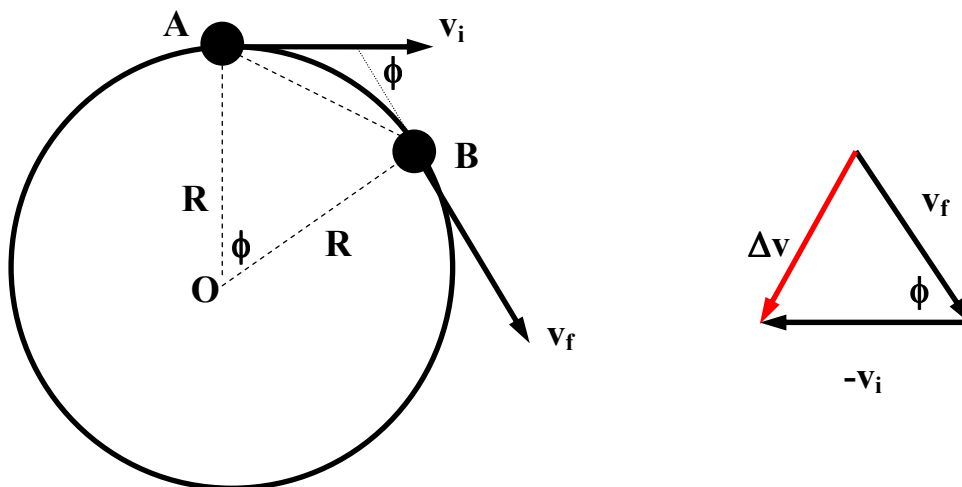
Example #2: In the Olympics, the Hammer Throw competition involves athletes spinning a heavy weight on a cord around several times before releasing the weight at an angle to the ground in order for it to travel a maximum distance. If one such weight of mass 15 kg was spun 4.5 times in 3.2 s in a circle of radius 1.7 m, then released at an angle of 37° to the ground,

- what was the average speed and acceleration of the weight as it was spun?
- how far was it flung, assuming it landed at about the same height it was released?

(see Circular Motion Ex 2 for answer)

Deriving these formulas for centripetal acceleration is somewhat tricky and not needed for the purposes of this unit. However, if you're mathematically inclined and love to torment your mind, the following information reveals the derivation of the first formula.

Identifying similar triangles will result in an expression for acceleration.



Note triangle **AOB** is similar to the above velocity triangle because they have the same vertex angle ϕ and both are isosceles.

If time t needed to move from **A** to **B** is small, chord **AB** \approx **arc AB**

\rightarrow therefore, $\frac{AB}{R} = \frac{\Delta V}{V}$ but distance **AB** = $(v_{av})(t)$

\rightarrow in this case, $v_{av} = v$ because only the direction changes

→substituting in, we get $\frac{vt}{R} = \frac{\Delta v}{v}$

→rearrange to obtain $\frac{v^2}{R} = \frac{\Delta v}{t} = \mathbf{a_c}$

→resulting in $\mathbf{a_c} = \frac{v^2}{R}$ where the direction of \mathbf{a} is on the radius.